AP Calculus BC

WS 85 Convergence of Series 2

1) $\frac{2}{5} + \frac{6}{20} + \frac{18}{80} + \frac{54}{320} + \dots$

$S = \frac{2/5}{1 - 3/4} = \frac{2/5}{1/4} = \frac{8}{5}$

2) I. $\sum_{n=1}^{\infty} \frac{3}{n}$ diverges p-series

II. $\sum_{n=1}^{\infty} \frac{n+1}{n+4}$ diverges by n^{th} term

$\lim_{n\to\infty} \frac{n+1}{n+4} = 1$

III. $\sum_{n=1}^{\infty} \left(\frac{-2}{(-5)^n}\right)$ converges by GST

$|r| = \frac{1}{5} < 1$

3) $\sum_{n=1}^{\infty} \frac{5n-2}{n(3^n)} = S$

LCT

$\sum_{n=1}^{\infty} \frac{1}{3^n}$ converges by GST

$\lim_{n\to\infty} \left[\frac{5n-2}{n(3^n)} \cdot 3^n\right] = 5 > 0$

S converges

4) $\sum_{n=1}^{\infty} \frac{2n}{3n^2+1} = S$

LCT

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series

$\lim_{n\to\infty} \left[\frac{2n}{3n^2+1} \cdot n\right] = \frac{2}{3} > 0$

S diverges

5) $\sum_{n=1}^{\infty} \frac{5^n}{3n^2+1} = S$

$\lim_{n\to\infty} \frac{5^n}{3n^2+1} = \infty$

S diverges by n^{th} term test

6) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} = S$

$\int_2^{\infty} \frac{1}{x(\ln x)^2} dx = \lim_{b\to\infty} \int_2^b \frac{1}{x}(\ln x)^{-2} dx$

$\lim_{b\to\infty} \left[-(\ln x)^{-1}\right]_2^b = \lim_{b\to\infty} \left[-\frac{1}{\ln b} + \frac{1}{\ln 2}\right]$

$= \frac{1}{\ln 2}$

S convers by Integral Test

7) $\sum_{n=1}^{\infty} \frac{\sin n}{3^n} = S$

AST

$+ \lim_{n\to\infty} \frac{1}{3^n} = 0$

$+$ dec.

$\sum \frac{1}{3^n}$ converges by GST

S converges absolutely

8) $\sum_{n=1}^{\infty} \frac{n(3^n)}{n!} = S$

Ratio Test

$\lim_{n\to\infty} \left[\frac{(n+1)(3^{n+1})}{(n+1)!} \cdot \frac{n!}{n(3^n)}\right]$

$\lim_{n\to\infty} \frac{3}{n} = 0 < 1$

S converges by Ratio Test

9) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^4+1} = S$

$\displaystyle\int_1^{\infty} \frac{x}{x^4+1}\,dx$

$\displaystyle\lim_{b\to\infty} \int_1^{b} \frac{x}{(x^2)^2+1}\,dx$

$\displaystyle\lim_{b\to\infty} \left[\frac{1}{2}\arctan(x^2) \right]_1^b$

$\displaystyle\lim_{b\to\infty} \left[\frac{1}{2}\arctan b^2 - \frac{1}{2}\arctan 1 \right]$

$\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{2}\left(\frac{\pi}{4}\right)$

$\frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8}$

S converges

10) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^4+1} = S$

LCT

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by p-series

$\displaystyle\lim_{n\to\infty} \left[\frac{n}{n^4+1} \cdot n^3 \right] = 1 > 0$

S converges by LCT

11) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n} = S$

AST

- $\displaystyle\lim_{n\to\infty} \frac{1}{3n} = 0$
- $\frac{1}{3}, \frac{1}{6}, \frac{1}{9} \dots$

$\sum \frac{1}{3n} = \frac{1}{3}\sum \frac{1}{n}$ diverges

S converges conditionally

12) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+3}{3n^2+4} = S$

AST

- $\displaystyle\lim_{n\to\infty} \frac{2n+3}{3n^2+4} = 0$
- $\frac{5}{7}, \frac{7}{16}, \frac{9}{31},$

LCT

$\sum \frac{1}{n}$ diverges by p-series

$\displaystyle\lim_{n\to\infty} \left[\frac{2n+3}{3n^2+4} \cdot n \right] = \frac{2}{3}$

S converges conditionally